Spin precession caused by spin-spin interaction between bounded electrons in quantum dots

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In this paper, we study the spin-spin interaction between two electrons bounded in a quantum dot. The result shows that spin-spin interaction will cause a pair of spins precessing synchronously. If the two spins are parallel at initial time, the total spin oscillates as a cosine function. If the two spins are antiparallel at initial time, the total spin keeps zero. The precessing period is proportional to the cube of quantum dot radius. For a 2D round quantum dot with radius $R = 50$nm, the precessing period equals nearly 12μs.

PACS numbers: 72.25.Rb , 03.65.Ge

I. INTRODUCTION

The spin dynamics in quantum dots attracts much attention in recent years, because the experiments indicate long spin relaxation time [1][2][3] in some kinds of quantum dots. This makes the quantum dot the promising candidate for qubit, which is the foundation of quantum computer. Many plans have been proposed for representing qubit and building unitary gates in quantum dots system. Some authors have researched the spin filling in a quantum dot [4] and proposed the qubit encoded by more than one electron spin, such as encoding a qubit in singlet and triplet states for two electrons in one dot or in two coupled dots [5][6]. Experiments have shown the singlet-triplet transition can be electrically controlled [1][7][8]. Furthermore, there are proposals for encoding a qubit in two specific spin states of three electrons in single dot [9]. In order to get a good representation of qubit and the corresponding gate sequences, it is important to understand the interaction on spins in quantum dots. Hyperfine interaction between the nucleus and electrons has been considered as the most important interaction on spins in some kinds of quantum dots [10][11]. It has been investigated by many authors [12][13][14][15].

The Coulomb interaction between bounded electrons has also been intensively studied. But according to quantum electrodynamics (QED), Coulomb interaction is only the non-relativistic approximation of electron-electron interaction. Considering the relative nature of electron movement, spin-spin interaction, which can be regarded as the interaction between two magnetons, should be included. There is great difference between Coulomb interaction and spin-spin interaction, because the latter can exert forces on spins. Considering spin-spin interaction, only the total angular momentum conserves. Spin-spin interaction has been researched in condensed matter physics after the QED theory was discovered. Overhauser [16] in his early articles has discussed the spin relaxation caused by spin-spin interaction between Bloch electrons. He found that the effect was much smaller than that caused by spin-orbit interaction and could be ignored. But in a quantum dot, the spin-orbit coupling is efficiently suppressed, while spin-spin interaction is enhanced because electrons are much closer to each other. On the other hand, the experiment about spin filling shows that there exist unpaired electrons in a multi-electron quantum dot. Then there exists the spin-spin scattering process caused by spin-spin interaction.

In quantum dots, spin-spin interaction does not cause spin relaxation or dephasing. But it has non-trivial influence to the evolution of spins in quantum dots. Therefore it is obliged to study the effects of spin-spin interaction.

The paper is organized as follows: in Sec. II we give the Hamiltonian of spin-spin interaction from QED theory. Then we discuss the selection rule of this interaction in Sec. III. In Sec. IV and Sec. V, we study the evolution process of two parallel and antiparallel spins respectively. Sec. VI is a brief conclusion.

II. SPIN-SPIN INTERACTION

The Hamiltonian of QED theory can be expressed as

$$H = \sum_s \int d^3\mathbf{p}\omega_{\mathbf{p}} c^\dagger_{\mathbf{p},s} c_{\mathbf{p},s} + \sum_\lambda \int d^3\mathbf{k}(-g_{\lambda\lambda}|\mathbf{k}|) a^\dagger_{\mathbf{k},\lambda} a_{\mathbf{k},\lambda} - e\int d^3\mathbf{r}\bar{\psi}\gamma^\mu\psi A_\mu. \quad (1)$$

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Here we have omitted the terms creating or annihilating positrons. The Fourier decompositions of the field operators are

$$\begin{aligned}\psi(\mathbf{r}) = &\quad \sum_s \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} \sqrt{\frac{m}{\omega_\mathbf{p}}} c_{\mathbf{p},s} u(\mathbf{p},s) e^{i\mathbf{pr}}, \\ A^\mu(\mathbf{r}) = &\quad \int \frac{d^3\mathbf{k}}{\sqrt{2|\mathbf{k}|}(2\pi)^3} \sum_\lambda (a_{\mathbf{k},\lambda}\epsilon^\mu(\mathbf{k},\lambda) e^{i\mathbf{kr}} + a^\dagger_{\mathbf{k},\lambda}\epsilon^\mu(\mathbf{k},\lambda) e^{-i\mathbf{kr}}). \end{aligned} \tag{2}$$

By using the canonical transformation and noticing the relation $\sum_\lambda g_{\lambda\lambda}\epsilon_\mu(\mathbf{k},\lambda)\epsilon_\nu(\mathbf{k},\lambda) = g_{\mu\nu}$, one can get an effective electron-electron interaction

$$V_{eff} = \sum_{s,s',s'',s'''} \int d^3\mathbf{p} d^3\mathbf{p}' d^3\mathbf{k} \frac{e^2}{2(2\pi)^3|\mathbf{k}|^2} \bar{u}(\mathbf{p}+\mathbf{k},s)\gamma^\mu u(\mathbf{p},s')\bar{u}(\mathbf{p}'-\mathbf{k},s''')\gamma_\mu u(\mathbf{p}',s'') c^\dagger_{\mathbf{p}+\mathbf{k},s} c_{\mathbf{p},s'} c^\dagger_{\mathbf{p}'-\mathbf{k},s'''} c_{\mathbf{p}',s''}. \tag{3}$$

Here we have used the low energy approximation $\omega_\mathbf{p} \approx m$. There are two terms in this effective Hamiltonian which exert forces on the spins. The first is the spin-spin interaction which is the coupling between one electron spin and the magnetic field caused by another electron spin. And the second is the spin-current interaction which is the coupling between the magnetic moment of one electron and the magnetic field produced by the translational motion of another. The spin-current interaction between bounded electrons can be neglected. The spin-spin interaction can be divided into two terms

$$V_{s-s} = V_D + V_{ex}. \tag{4}$$

The first term V_D represents flip of a pair of parallel spins, and can be called parallel interaction. The second term V_{ex} represents flip of a pair of antiparallel spins, and can be called antiparallel interaction. According to Eq. (3), the parallel interaction can be expressed as

$$V_D = \int d^3\mathbf{k} d^3\mathbf{p} d^3\mathbf{p}' \frac{e^2\hbar^2 (k_x + ik_y)^2}{8m^2c^2\epsilon_0(2\pi)^3|\mathbf{k}|^2} c^\dagger_{\mathbf{p}+\mathbf{k},\downarrow} c^\dagger_{\mathbf{p}'-\mathbf{k},\downarrow} c_{\mathbf{p}',\uparrow} c_{\mathbf{p},\uparrow} + h.c., \tag{5}$$

where $\uparrow$ and $\downarrow$ denote the spin-up and spin-down states in z direction respectively. And $c_{\mathbf{p},\uparrow}$ denotes the annihilation operator of spin-up electrons. The antiparallel interaction can be expressed as

$$V_{ex} = \int d^3\mathbf{k} d^3\mathbf{p} d^3\mathbf{p}' \frac{-e^2\hbar^2 \left(|\mathbf{k}|^2 + k_z^2\right)}{4m^2c^2\epsilon_0(2\pi)^3|\mathbf{k}|^2} c^\dagger_{\mathbf{p}+\mathbf{k},\uparrow} c^\dagger_{\mathbf{p}'-\mathbf{k},\downarrow} c_{\mathbf{p}',\uparrow} c_{\mathbf{p},\downarrow}. \tag{6}$$

To study the interaction between bounded electrons, the Hamiltonian should be expressed in real space. Eq. (5) can be transformed into

$$V_D = \int d^3\mathbf{k} d^3\mathbf{r}_1 d^3\mathbf{r}_2 \frac{e^2\hbar^2 (k_x + ik_y)^2 e^{i\mathbf{k}(\mathbf{r}_1 - \mathbf{r}_2)}}{8m^2c^2\epsilon_0(2\pi)^3|\mathbf{k}|^2} \psi^\dagger_\downarrow(\mathbf{r}_1)\psi^\dagger_\downarrow(\mathbf{r}_2)\psi_\uparrow(\mathbf{r}_2)\psi_\uparrow(\mathbf{r}_1) + h.c.. \tag{7}$$

The $\mathbf{k}$-integral is difficult to calculate for arbitrary $\mathbf{r}_1$ and $\mathbf{r}_2$ in 3D space. In this work, we study spin-spin interaction in a quasi-2D quantum dot sited in x-y plane. Then $\mathbf{r}_1 - \mathbf{r}_2$ sites in x-y plane at $z = 0$, and $\mathbf{k}$ can be integrated out from Eq. (7)

$$\int d^3\mathbf{k} \frac{(k_x + ik_y)^2 e^{i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2)}}{|\mathbf{k}|^2} = \frac{-2\pi^2 e^{2i\arg(\mathbf{r}_1 - \mathbf{r}_2)}}{|\mathbf{r}_1 - \mathbf{r}_2|^3}, \tag{8}$$

where $\arg(\mathbf{r}_1 - \mathbf{r}_2)$ is the angle between $\mathbf{r}_1 - \mathbf{r}_2$ and x axis. So the parallel interaction can be expressed as

$$V_D = \frac{-e^2\hbar^2}{32\pi m^2c^2\epsilon_0} \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \frac{e^{2i\arg(\mathbf{r}_1 - \mathbf{r}_2)}}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \psi^\dagger_\downarrow(\mathbf{r}_1)\psi^\dagger_\downarrow(\mathbf{r}_2)\psi_\uparrow(\mathbf{r}_2)\psi_\uparrow(\mathbf{r}_1) + h.c.. \tag{9}$$

This equation is valid only when the electrons are confined in a very thin layer. Accordingly, antiparallel interaction can be expressed as

$$\begin{aligned} V_{ex} = &-\frac{e^2\hbar^2}{2m^2c^2\epsilon_0} \int d^3\mathbf{r} \psi^\dagger_\uparrow(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r}) \\ &-\frac{e^2\hbar^2}{16m^2c^2\epsilon_0\pi} \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \psi^\dagger_\uparrow(\mathbf{r}_1)\psi^\dagger_\downarrow(\mathbf{r}_2)\psi_\uparrow(\mathbf{r}_2)\psi_\downarrow(\mathbf{r}_1). \end{aligned} \tag{10}$$

The antiparallel interaction conserves the total spin. From this point of view, it is similar to the exchange interaction. But there is difference between them. The exchange interaction need the overlap of wave functions, while antiparallel interaction need not.

III. SPIN-SPIN SCATTERING OF BOUNDED ELECTRONS IN QUANTUM DOTS

We consider a quasi-2D round quantum dot with radius R. The Hamiltonian is

$$H = H_0 + V_{s-s}, \tag{11}$$

where $H_0 = \sum_{\lambda,\sigma} \varepsilon_\lambda c^\dagger_{\lambda,\sigma} c_{\lambda,\sigma}$ is the bounded energy and V_{s-s} the spin-spin interaction. Because the number of electrons in quantum dots is definite, the Coulomb charging energy is a constant and is omitted. We assume the confining potential is zero inside the quantum dot and infinite outside it. It is well known that the bounded wave function can be expressed as Bessel functions in polar coordinates

$$\psi_{m,n}(\rho,\theta) = \frac{1}{R\sqrt{\pi} J_{|m|+1}(Z_{m,n})} J_m\left(\frac{Z_{m,n}\rho}{R}\right) e^{im\theta}, \tag{12}$$

where $m = 0, \pm 1, \cdots$ is the angular momentum in z direction and $Z_{m,n}$ the nth zero of Bessel function J_m. The bounded energy is

$$\varepsilon_{m,n} = \frac{\hbar^2}{2m^* R^2} Z^2_{m,n}. \tag{13}$$

The spin-spin interaction matrix is calculated

$$\begin{aligned}\langle m_3 n_3 \downarrow, m_4 n_4 \downarrow | V_D | m_1 n_1 \uparrow, m_2 n_2 \uparrow\rangle = -\frac{1}{4c^2}\int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \frac{e^{2i\arg(\mathbf{r}_1-\mathbf{r}_2)}}{|\mathbf{r}_1-\mathbf{r}_2|^3}\times \\ \times[\psi_4^*(\mathbf{r}_2)\psi_3^*(\mathbf{r}_1)\psi_1(\mathbf{r}_1)\psi_2(\mathbf{r}_2) - \psi_4^*(\mathbf{r}_1)\psi_3^*(\mathbf{r}_2)\psi_1(\mathbf{r}_1)\psi_2(\mathbf{r}_2)],\end{aligned} \tag{14}$$

where ψ_i denotes the wave function ψ_{m_i,n_i} in Eq. (12). We work in the atomic units, where $\hbar = e = m = 4\pi\epsilon_0 = 1$. By substituting Eq. (12) into Eq. (14), we obtain

$$\langle m_3 n_3 \downarrow, m_4 n_4 \downarrow | V_D | m_1 n_1 \uparrow, m_2 n_2 \uparrow\rangle = \frac{-\gamma_D}{4\pi^2 c^2 R^3}, \tag{15}$$

where $c = 1.370 \times 10^2$ is the light velocity and γ_D a coefficient which depends only upon the quantum numbers m and n of initial and final states. γ_D can be expressed as

$$\begin{aligned}\gamma_D \;=\; & \frac{1}{J_{|m_1|+1}(Z_1)J_{|m_2|+1}(Z_2)J_{|m_3|+1}(Z_3)J_{|m_4|+1}(Z_4)}\int_0^1 d\rho_1 d\rho_2 \int_0^{2\pi} d\theta_1 d\theta_2 \\ & \frac{\rho_1\rho_2 e^{2i\varphi} J_{m_1}(Z_1\rho_1)J_{m_2}(Z_2\rho_2)e^{i(m_1\theta_1+m_2\theta_2)}}{(\rho_1^2+\rho_2^2-2\rho_1\rho_2\cos(\theta_1-\theta_2))^{3/2}}[J_{m_3}(Z_3\rho_1)J_{m_4}(Z_4\rho_2)e^{-i(m_3\theta_1+m_4\theta_2)} \\ & -J_{m_3}(Z_3\rho_2)J_{m_4}(Z_4\rho_1)e^{-i(m_3\theta_2+m_4\theta_1)}],\end{aligned} \tag{16}$$

where $\varphi = \arg(\rho_1 e^{i\theta_1} - \rho_2 e^{i\theta_2})$ and $Z_i = Z(m_i, n_i)$. Above integral is calculated numerically by Monte Carlo method. The γ_D is nonzero only when m_1, m_2, m_3 and m_4 satisfy the relation

$$m_3 + m_4 - m_1 - m_2 = 2. \tag{17}$$

This can be explained by the conservation of total angular momentum. The spin-spin interaction satisfies the commutation relation $[V_D, L_z + S_z] = 0$. And the bounded states are the eigenstates of L_z and S_z. So the matrix element is not zero only when Eq. (17) is satisfied. We also find that γ_D satisfying the selection rule has the magnitude of $10^2 \sim 10^3$. For example, $\gamma_D = 3.823 \times 10^3$ when the initial and final states are $|01\uparrow, -11\uparrow\rangle$ and $|01\downarrow, 11\downarrow\rangle$ respectively.

The second term of antiparallel interaction in Eq. (10) denotes the interaction between two electrons at different position, which is proportional to $|r_1 - r_2|^{-3}$ and decreases sharply with the distance of two electrons increasing. The size of a quantum dot is usually much larger than the length unit of Bohr radius $0.529\mathring{A}$. So the second term is usually much smaller than the first term and can be neglected. The antiparallel interaction in atomic units is then

$$V_{ex} = \int d^3\mathbf{r} \frac{-2\pi}{c^2} \psi^\dagger_\uparrow(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r}). \tag{18}$$

The antiparallel interaction matrix is calculated

$$\langle m_3 n_3 \uparrow, m_4 n_4 \downarrow | V_{ex} | m_1 n_1 \uparrow, m_2 n_2 \downarrow \rangle = \int d^3\mathbf{r} \frac{-2\pi}{c^2} \psi_3^* (\mathbf{r}) \psi_4^* (\mathbf{r}) \psi_1 (\mathbf{r}) \psi_2 (\mathbf{r}). \tag{19}$$

Here we must consider the finite height of the quantum dot. The wave function in circular cylindrical coordinates is (choosing the lowest level in z direction [12])

$$\psi (\rho, \theta, z) = \sqrt{\frac{2}{a\pi}} \frac{1}{R J_{|m|+1}(Z_{m,n})} J_m \left(\frac{Z_{m,n}\rho}{R} \right) e^{im\theta} \sin(\pi \frac{z}{a}), \tag{20}$$

where a is the height of the quantum dot. Substituting Eq. (20) into Eq. (19), we get

$$\langle m_3 n_3 \uparrow, m_4 n_4 \downarrow | V_{ex} | m_1 n_1 \uparrow, m_2 n_2 \downarrow \rangle = \frac{-6\gamma_{ex}}{c^2 R^2 a}, \tag{21}$$

where γ_{ex} is a coefficient depending only upon the quantum numbers m and n of initial and final states

$$\gamma_{ex} = \frac{\int_0^1 d\rho \rho J_{m_1} (Z_1 \rho) J_{m_2} (Z_2 \rho) J_{m_3} (Z_3 \rho) J_{m_4} (Z_4 \rho)}{J_{|m_1|+1} (Z_1) J_{|m_2|+1} (Z_2) J_{|m_3|+1} (Z_3) J_{|m_4|+1} (Z_4)}. \tag{22}$$

Due to the conservation of angular momentum, the γ_{ex} is not zero only when next selection rule is satisfied

$$m_1 + m_2 - m_3 - m_4 = 0. \tag{23}$$

The γ_{ex} satisfying the selection rule has the magnitude of 10^0. For example, $\gamma_{ex} = 0.718$ when the initial and final states are $|01 \uparrow, -11 \downarrow\rangle$ and $|-11 \uparrow, 01 \downarrow\rangle$ respectively.

IV. THE EVOLUTION OF TWO PARALLEL SPINS

We propose a quantum dot with two parallel spins along z direction. In experiments, this can be realized by successively injecting two spins into a quantum dot with even number of electrons [1][5]. Electrons in two highest levels are unpaired, while the other levels are occupied by paired spins. So one can assume there are only two electrons, occupying two nearby levels. The spins will flip together due to parallel interaction, when the final and initial states are degenerate and the transition satisfies the selection rule of Eq. (17). If the initial and final states are non-degenerate, the energy difference is usually much larger than the spin-spin interaction energy. So the non-degenerate transition rate is very small and can be ignored. We assume the spins are up at initial time. Arbitrary initial state $|m_1 n_1 \uparrow, m_2 n_2 \uparrow\rangle$ has at most three degenerate states $|-m_1 n_1 \downarrow, m_2 n_2 \downarrow\rangle$, $|m_1 n_1 \downarrow, -m_2 n_2 \downarrow\rangle$ and $|-m_1 n_1 \downarrow, -m_2 n_2 \downarrow\rangle$. They do not always satisfy the selection rule of Eq. (17). Only when $|m_1|$ and $|m_2|$ are consecutive numbers or one of them is equal to -1, there exists one final state satisfying the selection rule. Below is a table showing the quantum numbers m and n of several levels sorted by energy. From this table, we can find the initial states permitting spin flip.

level	1	2	3	4	5	6	7	8	9
m	0	1	-1	2	-2	0	3	-3	1
n	1	1	1	1	1	2	1	1	2
energy($\frac{\hbar^2}{2m^* R^2}$)	5.783	14.68	14.68	26.37	26.37	30.47	40.71	40.71	49.22

For example, the degenerate state of the initial state $|01 \uparrow, -11 \uparrow\rangle$ is $|01 \downarrow, 11 \downarrow\rangle$. In this situation, the initial and final states generate the complete bases. Then the Hamiltonian can be written in a matrix

$$H = H_0 + V_D = \begin{bmatrix} E & V_D \\ V_D & E \end{bmatrix}, \tag{24}$$

where $E = (Z_{m1,n1}^2 + Z_{m1,n1}^2)/(2m^* R^2)$ is the total bounded energy in atomic units. And

$$V_D = \langle 01 \downarrow, 11 \downarrow | V_D | 01 \uparrow, -11 \uparrow\rangle = \frac{-\gamma_D}{4\pi^2 c^2 R^3} \tag{25}$$

is the parallel interaction energy. The evolution matrix evaluates

$$U(t) = e^{-iHt} = e^{-iEt} \begin{bmatrix} \cos V_D t & -i \sin V_D t \\ -i \sin V_D t & \cos V_D t \end{bmatrix}. \tag{26}$$

The time-dependent state can be expressed as

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle = e^{-iEt} \cos V_D t |\uparrow\uparrow\rangle - ie^{-iEt} \sin V_D t |\downarrow\downarrow\rangle, \tag{27}$$

where the quantum number m and n are omitted. The total spin at z direction is oscillating $S_z(t) = \cos 2V_D t$. The spins in a quantum dot will precess even without external magnetic field or nuclear spins. The precession frequency V_D/π depends on the size of the quantum dot, being inversely proportional to cube of radius. For example, in a quantum dot with radius $R = 50$nm the precession period is $T = \pi/V_D = 12.4\mu$s. When it is much shorter than the spin relaxation time, the precession can be observed. In GaAs/AlGaAs 2DEG quantum dots, it is impossible to observe the precession because the spin relaxation time is only several nanoseconds due to hyperfine interaction [10]. In a quantum dot made of the material with zero nuclear spin (such as Si), the spin relaxation time is much longer. Then it maybe possible to observe this precession.

A weak vertical magnetic field depresses hyperfine interaction and increases the spin relaxation time, but it also destroys the precession caused by parallel interaction. Because the degenerate transition becomes the non-degenerate transition, whose rate is much smaller, when a magnetic field is added. But if the magnetic field is so strong that the Zeeman splitting is comparable with the level spacing, the non-degenerate transitions may become degenerate. And the strong field will increase the spin relaxation time. The electron's wave function is more compact in the presence of magnetic field, that increases the spin-spin interaction. So the influence of a strong magnetic field to the precession is complicated and will be studied in future.

V. THE EVOLUTION OF TWO ANTIPARALLEL SPINS

Next we discuss the precession of two antiparallel spins in the antiparallel interaction V_{ex}. We propose a quantum dot with two antiparallel spins. At initial time, the two spins occupy two nearby levels $|m_1 n_1 \uparrow, m_2 n_2 \downarrow\rangle$. There are at most seven degenerate final states, $|-m_1 n_1 \uparrow, m_2 n_2 \downarrow\rangle$, $|m_1 n_1 \uparrow, -m_2 n_2 \downarrow\rangle$, $|-m_1 n_1 \uparrow, -m_2 n_2 \downarrow\rangle$, $|m_1 n_1 \downarrow, m_2 n_2 \uparrow\rangle$, $|-m_1 n_1 \downarrow, m_2 n_2 \uparrow\rangle$, $|m_1 n_1 \downarrow, -m_2 n_2 \uparrow\rangle$, $|-m_1 n_1 \downarrow, -m_2 n_2 \uparrow\rangle$. But the only one permitted by the selection rule of Eq. (23) is $|m_1 n_1 \downarrow, m_2 n_2 \uparrow\rangle$. This transition is a degenerate transition for arbitrary initial state. The Hamiltonian of electrons is

$$H = \begin{bmatrix} E & V_{ex} \\ V_{ex} & E \end{bmatrix}, \tag{28}$$

where $V_{ex} = \dfrac{-6\gamma_{ex}}{c^2 R^2 a}$ is the antiparallel energy. The diagonal term of antiparallel interaction is omitted because it is much smaller than E. The evolution matrix evaluates

$$U(t) = e^{-iEt} \begin{bmatrix} \cos V_{ex} t & -i \sin V_{ex} t \\ -i \sin V_{ex} t & \cos V_{ex} t \end{bmatrix}. \tag{29}$$

The state at time t is expressed as

$$|\Psi(t)\rangle = e^{-iEt} \cos V_{ex} t |\uparrow\downarrow\rangle - ie^{-iEt} \sin V_{ex} t |\downarrow\uparrow\rangle. \tag{30}$$

The spins precess synchronously. The precession frequency $V_{ex}/2\pi$ depends on the size of the quantum dot, being inversely proportional to the volume of the quantum dot. For a quantum dot with the radius of $R = 50$nm and the height of $a = 5$nm, the precession period is $T = 56\mu$s when the initial state is $|01 \uparrow, -11 \downarrow\rangle$. A magnetic field along z direction will not destroy the precession since the initial and final states keep degenerate.

VI. CONCLUSION

We have studied spin-spin interaction between two bounded electrons in a quantum dot. The interaction Hamiltonian includes two terms, the parallel interaction and antiparallel interaction, which are shown in Eq. (9) and Eq. (10) respectively. They cause two parallel or antiparallel spins to precess synchronously. If the two spins are parallel, and $|m_1|$ and $|m_2|$ are consecutive numbers or one of them is -1, the total spin oscillates with a period proportional to the cube of radius of the dot. A weak magnetic field along z direction will destroy this precession. If the two spins are antiparallel and occupy different levels, they always precess synchronously. The period is proportional to the volume of the dot.

VII. ACKNOWLEDGEMENT

This work was supported by the Natural Science Foundation 10274069, 60471052 and 16225419; the Zhejiang Provincial Natural Foundation M603193.

[1] Toshimasa Fujisawa, David Guy Austing, YasuhiroTokura, Yoshiro Hirayama, Seigo Tarucha, Nature **419**, 278 (2002)
[2] R. Hanson, B. Witkamp, L. M. K. Vandersypen, L.H. Willems van Beveren, J. M. Elzerman and L. P. Kouwenhoven, Phys. Rev. Lett. **91**, 196802 (2003)
[3] A. C. Johnson, J. R. Petta, J. M. Taylor, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson and A. C. Gossard, Nature **435**,925 (2005)
[4] L. H. Willems van Beveren, R. Hanson, I. T. Vink, F. H. L. Koppens, L. P. Kouwenhoven, L. M. K. Vandersypen, New Journal of Physics, **7**, 182 (2005)
[5] J. R. Petta, A. C. Johnson, J. M. Taylor, E. A. Laird, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, A. C. Gossard, Science **309**, 2180 (2005)
[6] Hans-Andreas Engel, Daniel Loss, science, **309**, 586 (2005)
[7] S. Tarucha, D. G. Ausing, Y. Tokura, W. G. van der Wiel, L. P.Kouwenhoven, Phys. Rev. Lett. **84**, 2485 (2000)
[8] F. H. L. Koppens, J. A. Folk, J. M. Elzerman, R. Hanson, L. H. Willems van Beveren, I. T. Vink, H. P. Tranitz, W. Wegscheider, L. P. Kouwenhoven, L. M. K. Vandersypen, Science, **309**, 1346 (2005)
[9] Jordan Kyriakidis, Stephen J. Penney, Phys. Rev. B **71**, 125332 (2005)
[10] I. A. Merkulov, A1. L. Efros and M. Rosen, Phys. Rev. B **65**, 205309 (2002)
[11] Igor Zutic, Jaroslav Fabian, S. Das Sarma, Rev. Mod. Phys.**76**, 323 (2004)
[12] Sigurdur I. Erlingsson, Yuli V. Nazarov, and Vladimir I. Fal'ko, Phys. Rev. B **64**,195306 (2001)
[13] Sigurdur I. Erlingsson, Yuli V. Nazarov, Phys. Rev. B **66**, 155327 (2002)
[14] Alexander V. Khaetskii, Daniel Loss, and Leonid Glazman, Phys. Rev. Lett. **88**, 186802 (2002)
[15] Y. G. Semenov, K. W. Kim, Phys. Rev. B **67**, 073301 (2003)
[16] Albert W. Overhauser, Phys. Rev. **89**,689 (1953)